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SYSTEMIC RISK: CLUSTERING AND CONTAGION MECHANISMS

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We propose a general framework to capture both contagion and clustering mechanisms arising in financial networks when balance sheet linkages across entities exist. Building on Eisenberg and Noe (2001), we develop a multiperiod clearing payment system, where the financial network evolves stochastically over time. We model explicitly the impact of default events on the state of the network and introduce a novel mathematical structure, the systemic graph, to measure the contagion and systemic effects propagating in the network over time. Numerically, we show that domino effects appear when the interbank liability structure is homogeneous, whereas clustering effects are noticeable when the structure is heterogeneous. Larger correlations between interbank liabilities reduce the domino contribution to systemic risk and increase default clustering, especially if liability exposures are highly volatile.

For bilateral exposures originating from derivatives trades, such as options, futures, and credit default swaps. Such trades expose each counterparty not only to market risk but also to counterparty risk. Indeed, through these linkages, distress or failure of a financial institution triggering large unexpected losses on its trades can seriously affect the financial status of its counterparties in the network, possibly leading them into default. The recursive interdependence in this network of exposures is typically referred to as *systemic risk*, and has been responsible for many failures and credit quality deteriorations experienced by banks during the crisis. (See also Capponi 2012 and Capponi and Larsson 2012 for more details.)

In stable times, the behavior of the network does not exhibit any anomalous behavior. However, in times of financial distress, the recent crisis has demonstrated that default events originating in a specific area of the network may propagate wider in the financial system and affect zones that were not considered particularly vulnerable to a given adverse scenario. Such an intricate structure of linkages can

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be naturally captured by using a network representation of the financial system. Starting with the seminal paper by Allen and Gale (2001), who employed an equilibrium approach to model the propagation of financial distress in a credit network, many other approaches have been proposed to explain this phenomenon. Gai and Kapadia (2010) use statistical techniques from network theory to model how contagion spreads via counterparty exposures. Battiston et al. (2012a) describe the time evolution of the interbank network, and introduce the financial accelerator to characterize the feedback effect arising from changes in the financial conditions of an agent. Battiston et al. (2012b) demonstrate that the systemic risk does not necessarily decrease if the connectivity of the underlying financial network increases. Cifuentes, Ferrucci, and Shin (2005) show that the effects of financial distress at some financial institutions can force other financial entities to write down the value of their assets, and this may consequently trigger other defaults. Using a static approach, Amini, Cont, and Minca (2011) analyze default contagion and short term counterparty risk in the context of interbank lending, using tools from random graph theory. Reduced form models of dynamic contagion are instead considered in Dai Pra et al. (2009) and Dai Pra and Tolotti (2009), and most recently by Cvitanic, Ma, and Zhang (2010) and Giesecke, Spiliopoulos, and Sowers (2011). Capponi and Larsson (2012) analyze the systemic risk associated with default of a company via the interplay between equilibrium behavior of investors, risk preferences, and cyclicality properties of the default intensity.

We propose a novel framework aimed at capturing both *clustering* and *contagion* mechanisms arising from balance sheet interactions across entities. Our framework builds on the approach proposed by Eisenberg and Noe (2001), who develop a fairly general model of a clearing system, and then analyze the systemic effects of the default of an entity on its counterparties. Differently from Eisenberg and Noe (2001), who consider a static one-period model, we allow for stochastic dynamics to describe the time evolution of the financial network, and model the impact of default events on the state of the network. We allow for *multiple clearing* dates, with clearing payments satisfying the standard conditions imposed by bankruptcy laws in each date (limited liability of equity, absolute priority requirements, and proportional repayments of liabilities in default). In each time period, the financial system is modeled as a digraph, where nodes represent entities and edges liability relations between them.

Although our framework accommodates any liability structure, we specialize it to the case where such liabilities consist of call options in order to capture the impact of volatility on interbank liabilities, a relevant driver of counterparty risk propagation in financial markets. As option instruments are highly sensitive to volatility, the resulting analysis can provide systemic risk indicators in financially distressed environments. We further remark that options represent a sizable component of the total liabilities of an institution, and must be listed on the balance sheet according to the accounting classification requirements in Financial Accounting Standards Boards (2007).

Each node is associated with assets and operating cash inflows that the underlying entity possesses, and can be active or defaulted. The state of network is fully characterized by the state of all nodes, as well as by the interbank liability structure. We introduce a novel mathematical structure, the *systemic graph*, which provides a complete representation of the clustering and contagion effects within the network. The vertices of such a graph are called clusters and can be of two types, *source* and *contaminated*. Each cluster identifies an area of the network consisting of simultaneously defaulting nodes, which influenced each other because of direct or indirect linkages in the underlying liability graph. The source clusters represent the triggering components of systemic failures. The contaminated clusters, instead, identify areas of the financial network that defaulted because of the dynamic consequences implied from previous failures in other areas of the network.

The systemic graph represents a useful tool for systemic risk analysis, as it allows to fully track default cascades. Given a source cluster, each directed path originating from it identifies a path of systemic failure, which may then be closely monitored by a regulator wishing to take precautionary measures to prevent systemic crises. By a numerical analysis summarized by the systemic graph, we show that a *default cascade* is more frequent when the interbank liability structure is homogeneous; that is, the amounts of liabilities between any pair of nodes are of similar size. On the contrary, when the network is heterogeneous, default events tend to *cluster*, that is, to occur simultaneously, given that the reduced payments coming from defaulted entities have a stronger impact on the solvency state of the remaining entities. Moreover, higher volatility in exposures and interbank correlations exacerbate simultaneous occurrence of defaults, and result in large clusters of defaulted nodes.

The rest of the paper is organized as follows. Section I provides the framework. Section II develops the multi-period clearing system model. Section III develops measures of systemic risk for the network. Section IV concludes the paper.

I. FRAMEWORK

The financial network is modeled as a digraph G = (V, E), where the set V of nodes represents the financial firms, and the set E of edges the liability relations between nodes (a directed edge between node *i* and *j* indicates that *i* is a debtor of node *j*). We consider a finite time horizon, which is divided into discrete intervals, [t, t + 1), that are indexed by $t \in \{0, 1, \ldots, T\}$. The financial system is fully characterized by the following quantities associated to node $i \in V$ and edge $(i, j) \in E$ in each time *t*.

$$\mathbf{L}^{t} \in \mathbb{R}_{\geq 0}^{n \times n}, L_{ij}^{t}: \text{liability node } i \text{ owes to } j \text{ at time } t$$

$$\mathbf{I}^{t} \in \mathbb{R}_{\geq 0}^{n}, l_{i}^{t}: \text{ total liabilities node } i \text{ owes to other nodes at time } t.$$

$$l_{i}^{t} = \sum_{j \neq i, j \in V} L_{ij}^{t}.$$

$$\mathbf{\Pi}^{t} \in \mathbb{R}_{[0,1]}^{n \times n}, \Pi_{ij}^{t}: \text{liability of } i \text{ to } j \text{ as a proportion of } i \text{ 's total liabilities at time } t.$$

$$\Pi_{ij}^{t} = \frac{L_{ij}^{t}}{l_{i}^{t}} \text{ if } l_{i}^{t} > 0 ; 0 \text{ otherwise.}$$

 $\mathbf{p}^t \in \mathbb{R}^n_{\geq 0}$, p_i^t : payments node *i* makes to other nodes at time *t*. $t^t \in \mathbb{R}^n_{>0}$, t_i^t : operating cash inflow node *i* has at time *t*. $\mathbf{c}^t \in \mathbb{R}^n_{\geq 0}$, c_i^t : cash node *i* owns at time *t*.¹

Both liabilities and operating cash inflows are modeled via stochastic processes. As mentioned in the introduction, the term structure of liabilities is assumed to consist of options with different expirations, sold by each node of the network to any other at initial time. We denote the underlying asset on which the call option sold by *i* to *j* is written by x'_{ij} and denote the corresponding strike price by K'_{ij} . Further, the number of options is denoted by N'_{ij} . We then have that the liability *i* owes to *j* at time *t* is:

$$L_{ij}^{t} = N_{ij}^{t} (x_{ij}^{t} - K_{ij}^{t})^{+}.$$

We also impose that all nodes have positive operating cash inflow for all times; that is, for each *i*, t_i^t is an almost surely positive stochastic process, $t \in \{0, 1, ..., T\}$. This is one of the simplest sufficient conditions needed to guarantee the uniqueness of clearing payments in the single-period model developed in Eisenberg and Noe (2001). Such an assumption will be used when we prove the uniqueness of clearing payment sequence in Section II.

The time behavior of the financial network over time may be described by the pair (\mathbf{L}^{t}, t^{t}), $t \in \{0, 1, \ldots, T\}$, of stochastic processes. Given a time evolving financial network (\mathbf{L}^{t}, t^{t}), our objective is to model the propagation of defaults within the network, and provide effective measures to assess the systemic risk level.

II. MODEL

We develop a model for a multi-period multilateral clearing system, based on the framework described above. We start providing preliminary definitions that will be used subsequently in the paper.

The *cash* of node *i* at time *t*, denoted by c_i^t , is recursively defined as:

$$\begin{cases} c_i^0 = \sum_{j \neq i, i \in V} \prod_{j \in V}^0 \rho_j^0 + \iota_i^0 \\ c_i^t = \sum_{j \neq i, i \in V} \prod_{j \in V}^t \rho_j^t + \iota_i^t + (1+r)(c_i^{t-1} - l_i^{t-1})^+ & \text{for } t \in \{1, \dots, T\}, \end{cases}$$
(2.1)

where r is the market interest rate, assumed to be deterministic.

Definition 2.1. A node $i \in V$ defaults at time t if it cannot repay in full its liabilities due at t using his available cash, that is, $c_i^t < l_i^t$. The *default set* by the end of time t-1, denoted by D^t , includes all nodes, which defaulted by time $s \le t-1$. Clearly, $D^0 = \emptyset$.

^{1.} Strictly speaking, this also includes cash equivalents, that is, assets which are readily convertible into cash, such as money market holdings, short-term government bonds, or treasury bills.

A. Default Mechanism

Suppose a node *i* defaults by the end of time *t*. Then all liabilities owed by node *i* from t + 1 to *T* are due immediately, while the financial claims against other nodes from t + 1 to *T* cannot be realized yet. According to Chapter 7 under the bankruptcy laws of the United States, a bankruptcy trustee is appointed by the node *i*'s creditors to administer the bankruptcy estate. The trustee in general sells all the assets through an auction and distributes the proceeds to the creditors (see Bris, Welch, and Zhu 2006). In the context of our model, we assume that the trustee collects the payments that node *i* is supposed to receive from its debtors in the network, and distributes them to node *i*'s creditors after the assumed time horizon. Although a defaulted node is replaced by a bankruptcy trustee, mathematically, we continue to use the same notation of a defaulted node for the trustee; such a replacement has no impact on any of following computations.

B. Clearing Payment Sequence

Due to the presence of multiple clearing dates, we need to define a sequence of clearing payments, that is, payments that each node makes under a multi-period multilateral clearing system. Such a sequence also satisfies the standard conditions imposed by the bankruptcy law mentioned in Eisenberg and Noe (2001).

Definition 2.2. Given a time sequence $\{(\mathbf{L}^t, \iota^t)\}_{t=0}^T$, a time sequence of $\{\mathbf{p}^{t*}\}_{t=0}^T$ is a *clearing payment sequence* if it satisfies the following conditions:

a. *Payment less than liability*. The total payment $p_i^{t^*}$ node *i* makes is non-negative and does not exceed total liability l_i^t outstanding in each time period *t*:

$$0 \le p_i^{t*} \le l_i^t$$
 for $i \in V, t \in \{0, 1, ..., T\}$.

b. Proportional repayment of liabilities. A node $i \in V$ repays liabilities to one of his creditors according to a proportional mechanism; that is, node *i* pays $\prod_{ij}^{t} p_i^{t^*}$ to node *j* at time *t*.

c. *Absolute priority requirements*. In each time interval *t*, either node *i* repays in full its liabilities or, if it defaults, it uses all node *i*'s available cash to repay current creditors. Mathematically:

$$p_i^{t^*} = \mathbf{1}_{i \notin D^t} \min\{l_i^t, c_i^t\} \quad \text{for } i \in V, t \in \{0, 1, \dots, T\}$$
(2.2)

Lemma 2.3. In case of a single clearing date, the clearing payment sequence defined above coincides with the clearing payment vector in Eisenberg and Noe (2001), and it is unique.

Proof. For a single clearing date, $D^0 = \emptyset$ by definition. By equation (2.2):

$$p_i^{0} = \min\left\{ l_i^{0}, c_i^{0} = \sum_{j \neq i, j \in V} \prod_{j \neq i}^{0} p_j^{0} + \iota_i^{0} \right\} \text{ for } i \in V.$$

The solution is the clearing payment vector defined in Eisenberg and Noe (2001). As, by assumption, all nodes in the network have positive operating cash inflow, then by Eisenberg and Noe (2001), Theorem 2, the clearing vector is unique.

Lemma 2.4. Given a sequence of financial networks, there exists a unique clearing payment sequence.

Proof. We prove the above lemma by induction. At time 0, by Lemma 2.3, \mathbf{p}^0 exists and is unique. Suppose the statement to be true for time t - 1. At time t, since the remaining cash $(c^{t-1} - \mathbf{1}^{t-1}) \vee \mathbf{0}$ is determined from t - 1, we redefine $\iota^t + (1 + r)$ $(\mathbf{c}^{t-1} - \mathbf{1}^{t-1}) \vee \mathbf{0} = \tilde{\iota}^t$ and rewrite (2.2) as:

$$\begin{cases} p'_i = \min\left\{l'_i, c'_i = \sum_{j \neq i, j \in V} \prod'_{ji} p'_j + \tilde{\iota}'_i\right\} & \text{for } i \notin D\\ p'_i = 0 & \text{for } i \in D \end{cases}$$

By Eisenberg and Noe (2001) there exists a unique solution, \mathbf{p}^t , to the above system of equations. Thus, in each time interval there exists a unique payment vector, and hence a unique clearing payment sequence.

III. MEASURING SYSTEMIC RISK

A. Computation

Given a sequence of financial networks, Algorithm 1 given below recovers the clearing payment sequence and the associated sequence of default sets.

Algorithm 1: Default Propagation Algorithm.
1: procedure DEFAULT PROPAGATION $(\{(L', \iota')\}_{\iota=0}^T)$
2: for $t \leftarrow 0$ to T do
3: Solve (2.2) by the <i>fictitious default algorithm</i> ,
4: return \mathbf{p}^t , \mathbf{c}^t
5: $D^{t+1} \leftarrow D^t \cup \{i \in V \mid c_i^t < l_i^t\}.$
6: end for
7: end procedure

The algorithm uses the *fictitious default algorithm* proposed by Eisenberg and Noe (2001) as a subroutine. The latter is an efficient algorithm to recover the clearing payment vector at a given time. Using the fact that the fictitious default algorithm will take at most *n* steps to recover the clearing payment in each time interval, the proposed algorithm will terminate in n(T + 1) steps.

B. Systemic Graph and Measures

We measure the systemic risk across two dimensions: default cascades occurring across time (domino effect), and default clustering, that is, blocks of nodes defaulting on a fixed time. Before proceeding further, we review the concept of a strongly connected component of a graph. Given a graph G, a component C is said to be *strongly connected* if for any pair x, y of nodes in C, there exists at least a directed path from x to y in the subgraph induced by C, and all directed paths only cross nodes in C. Notice that, given two strongly connected components C_1 and C_2 , there exists no direct edge from a node in C_1 to a node in C_2 .

We construct an acyclic graph, called the *systemic graph*, which consists of clusters of two types: *source* and *contaminated*. Such clusters are obtained from the execution of the algorithm 1 as follows. Let $\{D^1, D^2, \ldots, D^T\}$ be the time sequence of default sets.

• t = 1. Denote by $\{C_1^1, C_2^1, \dots, C_i^1\}$ the clusters formed at t = 1. Each cluster $C_j^1 \subset D^1$ is a strongly connected component of the subgraph of *G* induced by D^1 . Each cluster C_j^1 , $1 \le j \le i$ is called a *source* cluster. Next we compute the list of all nodes *contaminated* by each cluster C_j^1 . A node *y* is contaminated by the cluster C_j^1 , if there exists at least one node in C_j^1 that has liabilities towards *y* at time 1.

• $t = k, k \ge 2$. Denote by $\{C_1^k, C_2^k, ..., C_l^k\}$ the clusters formed at t = k. If a cluster C_m^k contains at least one contaminated node, then its type becomes contaminated. If no node in the cluster C_m^k is contaminated, then the cluster is a source cluster. We insert a direct edge between C_l^h and C_l^k , h < k, if there exists a node in the cluster C_l^h , which was contaminated by a node in the cluster C_l^h .

In particular, notice that source clusters do not have incoming edges. Contaminated clusters, instead, must necessarily have incoming edges, but do not necessarily have outgoing edges. Some observations are in order. It may happen that two initially disconnected components of the systemic graph can later recombine into one component. If this happens at time t, it means that a cluster C_i^t is contaminated by two clusters C_j^s and C_k^s , formed at time s < t. The source clusters model the triggering components of the systemic failures and capture the clustering effect. The domino effect is measured by the maximum depth of the systemic graph (the higher the depth, the higher the effect).

C. Simulation Results

We provide an illustration of contagion and clustering effects captured by our framework. More specifically, we consider two different network configurations: (1) homogeneous interbank liabilities and (2) heterogeneous interbank liabilities. In the homogeneous case, liabilities between each pair of nodes are of similar size; in the heterogeneous case, each node always has higher netted liabilities towards lower indexed nodes (i.e., he needs to pays more than what he receives). Next, we present the results of Monte Carlo simulations under both scenarios. We consider a fully connected network of 40 nodes, and assume a time horizon T = 40, thus resulting in 40 payment periods. We fix the number of runs to 50. Throughout our simulations, we assume that the asset values underlying the call option liability follow geometric Brownian motions. We fix the strikes K_{ij}^{ij} to 10 for all t and $i, j \in V$; the number of

Table 1. Monte-Carlo Statistics Reporting Systemic Components of the Network.			
Mean Value of Measures	Homogeneous	Heterogeneous	
(1) Maximum depth of domino chain	19.02	5.10	
(2) Total number of defaulted nodes	32.84	40.00	
(3) Elapsed time before all defaults occur	31.54	5.60	
(4) Default propagating rate = $(2) / (3)$	1.05	7.54	

options each node holds, N_{ij}^t , is 1 for all *t* and *i*, $j \in V$. The operating cash in inflow that each node has at time 0 is $l_i^0 = 120$, for $i \in V$. Homogeneous and heterogeneous liability matrices are characterized by the initial value of the asset. In the homogeneous case, the asset at time 0 is given by:

$$x_{ij}^{0} = \begin{cases} 30 & \text{for } i, j \in V, i \neq j \\ 0 & \text{for } i, j \in V, i = j, \end{cases}$$

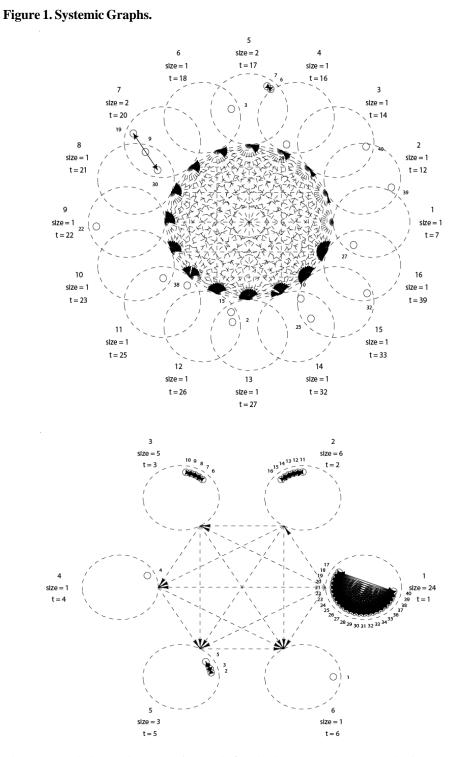
whereas in heterogeneous case, it is

$$x_{ij}^{0} = \begin{cases} 39 & \text{for } i, j \in V, i > j \\ 21 & \text{for } i, j \in V, i < j \\ 0 & \text{for } i, j \in V, i = j. \end{cases}$$

Systemic Graph. We start presenting the results obtained under a scenario where the asset values x_i^i 's are assumed to be uncorrelated, and the diffusion coefficients are the same, that is, $\mu_i = 0.1$ and $\sigma_i = 0.4$ for all *i*. The relevant statistics are provided in Table 1. We also report the systemic graph extracted from a snapshot of our simulations in Figure 1.

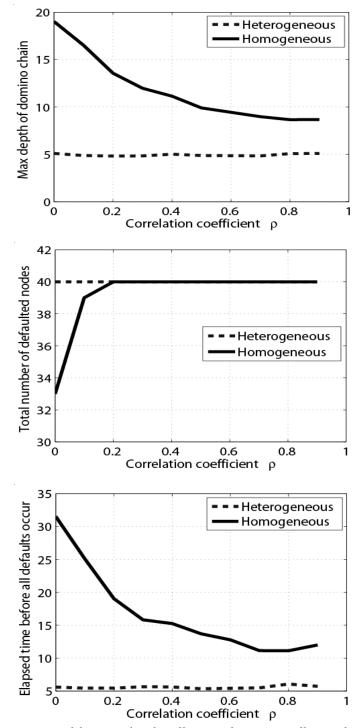
Both Table 1 and Figure 1 show that when the network has a homogeneous liability structure, default events happen in cascade. As all nodes are equally liable and creditors to each other, the failure of a node on a given date does not impact significantly the currently solvent nodes within the network. On the other hand, when the network is heterogeneous, defaults cluster. This is because the reduced payments coming from defaulted nodes have a stronger immediate impact on the solvency state of the others. Indeed, when node *N* defaults, node N - 1 will receive a reduced payment from *N* and will have to pay all other nodes more than what it receives. This will trigger its default immediately and propagate recursively to lower indexed nodes, thus resulting in clusters with larger size.

Such observed effects are consistent with empirical evidence provided in the academic literature by Angelini, Maresca, and Russo (1996), who analyzed the knock-on possibility within the Italian intraday netting system, and Furfine (2003), who considered the degree to which the failure of one bank would cause other failures in the federal fund market of the United States. A common theme of their results suggests that the scenario in which default events cluster depends on the systemic importance of the failing bank. We observe similar behaviors in our simulations, where in the homogeneous case none of the nodes is systemically

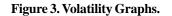


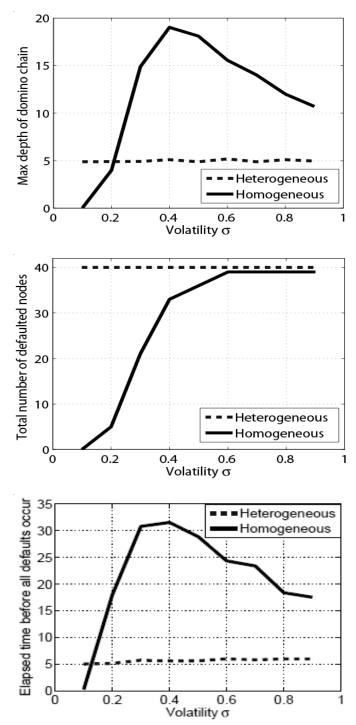
The top panel shows the systemic graph for the homogeneous case. The bottom panel shows the graph for the heterogeneous case. Arrows between nodes represent contamination relations.

Figure 2. Correlation Graphs.



All graphs are generated by assuming that all asset values are equally correlated with each other. The correlation parameter ρ is varied, while volatilities are always fixed to $\mu = 0.1$ and $\sigma = 0.4$.





All graphs are generated by assuming that all asset values are uncorrelated with each other, and have equal drift and volatility. We report the systemic behavior as a function of the volatility parameter σ .

more important than others, and consequently the failure of one node does not make significant knock-on impact on other nodes on the same date. On the other hand in the heterogeneous case, the larger liabilities owed by higher indexed nodes make them systemically more important than lower indexed nodes. Consequently they are able to trigger knock-on impact on the other nodes.

In practice, financial institutions with larger liabilities and size of exposures tend to be systemically more important according to the assessment methodology developed in Basel III (BCBS 2010). This is consistent with the results obtained in the heterogeneous case. Hence, they support the preventive measures against systemically important financial institutions suggested in Basel III (BCBS 2011), where such institutions are subject to higher capital requirements.

Another empirical study supporting our numerical results is the one conducted by Cont, Santos, and Moussa (2012). They show that the interbank Brazilian network exhibits a heterogeneous structure, both in terms of network connectivity and size of liability exposures. Indeed, there exists a strong positive correlation between the interbank liability size and the likelihood that default events cluster around a node. Their findings are consistent with results we obtain for the heterogeneous network structure, where we see that defaults of higher indexed node cluster at the earliest time due to their higher liabilities exposures.

They find that clusters of defaulted nodes are of small sizes and consist of systemically important nodes. In our numerical simulations, clusters are of larger size, most likely because we consider an extreme case where the network is fully connected, whereas the interbank Brazilian network may not be fully connected. We further remark that our framework captures not only the knock-on impact caused by the failing nodes but also the transmission effects propagating in cascade to the next dates.

Volatility and Correlation Effects. We analyze how volatility and correlation impact the systemic behavior of the network. Due to high heterogeneity of liability exposures in the heterogeneous configuration, both clustering and domino effects are mildly affected by increases in correlation or volatility. Differently, when the network is homogeneous, correlation increases will be associated with closer co-movements of the node's liability exposures and consequently result in larger default clustering (defaults are more likely to occur simultaneously), and shorter default cascades. This is clearly captured in Figure 2, where we also see that increases in correlation reduce the time before all defaults occur.

Figure 3 shows that when volatility is small, few nodes default if the network has a homogeneous liability structure. As the liabilities exhibit a small fluctuation around the initial values, the payments that each node will receive from creditors will be sufficient to repay debtors (also consider that each node has operating cash inflows to use). As volatility increases, the optionality embedded in the liability exposures will increase the risk, and consequently result in a larger number of defaults, and more noticeable domino effects. However, when the volatility becomes too large, the further amplification introduced by the optionality in interbank liabilities will make the network become more heterogeneous, and consequently result in a larger cluster and smaller domino effect.

IV. CONCLUSIONS

We developed a multi-period clearing payment system building on the approach originally proposed by Eisenberg and Noe (2001). Our framework is able to capture the systemic effects of default propagation within a financial network over a time horizon. We analyze both domino and clustering effects arising in the financial network. We have shown that there exists a unique clearing payment sequence and provided an algorithm to recover it. We introduced a novel object, the systemic graph, to precisely quantify the cascade and clustering phenomena appearing in the network.

In order to assess the behavior of the network in highly volatility environments, we specialized our framework to the case when the term structure of liabilities consists of call options. We numerically analyzed the clustering and domino effect in the network under two relevant cases, namely homogeneous and heterogeneous liability structures. Our results indicate that default cascades are common when interbank liabilities are homogeneous. On the contrary, when the network is heterogeneous, default events cluster as the reduced payments coming from defaulted entities have a stronger impact on the solvency state of the remaining entities. Higher correlations between interbank liabilities make the domino effect smaller, and default clustering higher. While small volatilities have a minor impact on the default status of the network, higher values will make simultaneous default occurrences more likely.

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